Wieringa Surface: The Implementation of Aperiodicity into Architectural Acoustics

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Abstract. At the intersection of order and disorder exists a balance that is referred to as aperiodicity. This mathematical concept is exemplified by the famous Penrose tiling methods, which display local five-fold symmetries that quickly dissipate globally. Architectural designers have previously exploited this property through floors tiling's, sculptures, and building facades. However, like mathematicians in the latter half of the 20th century such as Nicolaas Govert de Bruijn and R.M.A. Wieringa, designers in the early 21st century are now looking to the third dimension. In acoustics, surfaces with complex sectional profiles have been demonstrated to have scattering properties where the relationship between depth and width of undulations relate to the amount and frequency of the sound scattered. Mathematical sequences have been demonstrated to have optimal sound scattering properties, however, these mathematical sequences are periodic, and can therefore create unwanted acoustic effects. This research proposes combining the concept of aperiodicity with the predictability of a mathematical formula and defined through a parametric model, where the acoustic performance of these aperiodic surfaces will be analyzed through simulations. The contributions of this research are two-fold: first, we have developed a novel approach to creating a 3D object using parametric modeling to generate a Wieringa Surface, which when orthographically projected to 2D creates a Penrose Tiling; and second, we have carried out preliminary simulations that suggests there is a potential for this complex mathematical surface to exhibit sound scattering properties without periodic effects. This paper documents the parametrization of a specific case of the "Cut and Project Method", a mathematical method by which a slice of a multi-dimensional periodic space is selected through a cutting window and then projected to a lower dimension to produce aperiodic forms.

Keywords: Aperiodic Tiling · Mathematics and Architecture · Wieringa Surface · Penrose Tiling · Architectural Acoustics · Sound Scattering

1 Introduction

Architecture and acoustics share many common roots in mathematics. These disciplines rely on many of the same mathematical models and methods for the prediction, morphology, and optimization of behaviors. This is why the specialization of architectural

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acoustics utilizes mathematics to predict the auditory behavior of a space, explore new geometrical morphologies, and optimize acoustic-related scenarios. This paper documents the development of a parametric model for a mathematical surface that was observed by RMA Wieringa and described by N.G. de Bruijn. Although de Bruijn speculated about its applications in architecture, a supplementary hypothesis is presented for architectural acoustics. This approach was informed by previous works and observations made by Alan Mackay, while a formulated parametric model provides a novel computational approach to the established mathematical method. Iterations of the mathematical surface are tested through simulations and analyzed for performance. A case study consisting of three different geometrical scattering surfaces was conducted, where physical models for each of the surfaces was casted to demonstrate de Bruijn's initial architectural application ideas.

2 Background

2.1 Mathematical Definitions

In this section, we clarify all mathematical terminology used throughout this paper. "Euclidean space" is the vector space of all n-tuples of real numbers, $(x_1, \ldots, x_n)$. All geometries discussed within this paper reside in Euclidean space. When the word "space" is used singularly, then it refers to when $n = 3$, while the word "plane" is used for when $n = 2$. A "topological disc" refers to any set whose boundary is a single simple closed curve. A "plane tiling" is a countably infinite arrangement of closed topological disks on a plane without any gaps or overlaps. The individual components of a plane tiling are called "tiles". When the words tile(s) or tiling(s) are used singularly, then this refers to the aforementioned mathematical objects that reside on the plane unless otherwise specified. A tiling is "periodic" when there is an ordered copying and translation of a specific set of finite tiles. Thus, periodic tilings have translational symmetry, however they may, or may not, have rotational and/or reflection symmetry. Periodic tilings are said to exhibit "periodicity." If there is no periodicity, then there is no translational symmetry, and a tiling is said to be "non-periodic". In this paper, we will use the word "aperiodic" as a synonym for "non-periodic" and will use the two terms interchangeably. Aperiodic tilings are said to exhibit "aperiodicity"; however, aperiodicity can have alternate meanings within other mathematical contexts. A well-known example of aperiodicity is the Penrose tilings (Fig. 1) developed by the mathematical physicist and Nobel laureate, Sir Roger Penrose. (Penrose, 1974; Penrose, 1979).

2.2 The Fourier Transform and Aperiodicity

Mathematically, both periodic and aperiodic tessellations can be constructed in any dimension. In the field of crystallography however, tessellations of unit crystal cells were thought to only be periodic due to observed physical properties and experimental results. This imposed periodicity on structure meant that a crystal needed to be spatially arranged to follow the crystallographic restriction theorem which states that, if a set is periodic and the discrete group of translations of that set has more than one center

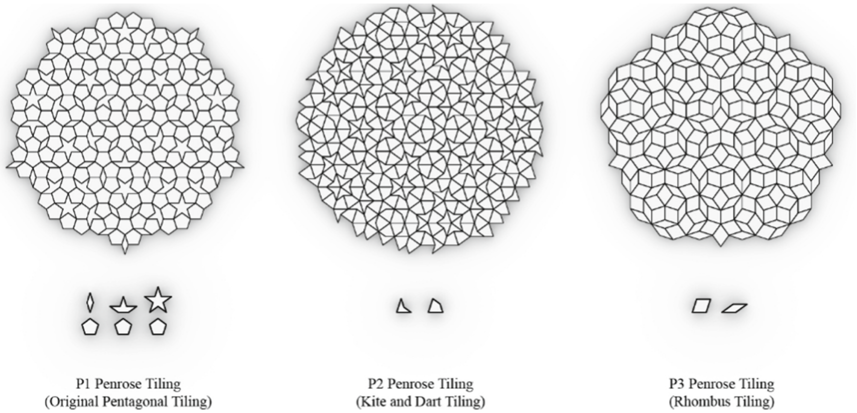

Fig. 1. Penrose Tiling variations

of rotation, then the only rotations that can occur are by 2, 3, 4, and 6. (Senechal, 2006) Theoretical crystallographer, Alan Mackay, (1981) was one of the first to consider spatial arrangements outside the periodical discourse. Using the vertices of Penrose's Rhombus tiling (P3), Mackay generated a sphere packing of both the plane and space and put them through a mathematical transformation known as the Fourier Transform (Fig. 2). The Fourier Transform decomposes a function and reformulates it into one that depends on spatial or temporal frequency. It is extensively used in the analysis of wavelike phenomena such as acoustical decomposition and diffraction patterning. (Bracewell, 1986) Mackay realized that his patterns displayed clear strong and weak modulations of the transform. There was a clearly defined ten-fold symmetry diffraction pattern after the transform was performed indicating that there is an ordering within the aperiodic arrangement despite the lack of translational symmetry. This construction allowed Mackay (1982) to theorize that arrangements of unit crystal cells could be both periodic and aperiodic. A year later at the National Bureau of Standards (now National Institute for Standard and Technology) in the city of Gaithersburg, Maryland in the USA, Mackay's theories were experimentally verified by Daniel Schectman's discovery of quasicrystals, a form of solid-state matter that exhibits aperiodicity. (Shechtman et al., 1984).

2.3 Architectural Acoustics

In architecture, the prediction of acoustic performance is complicated by the interaction of numerous competing factors, including space dimensions, proportions, geometry, material properties, and surface details. An important determinant of acoustical behavior is the design of architectural surfaces that affect the propagation of sound reflections through space. Such reflections may cause flutter echoes or comb filtering if they are not carefully controlled. (Long, 2014) In proper acoustical design, sound reflections can be effectively tamed by evenly dispersing the reflected sound energy. (Jaramillo and Steel, 2015) Several studies have examined the behavior of sound reflections off different surface geometries and how these reflections contribute to shaping the spatial auditory

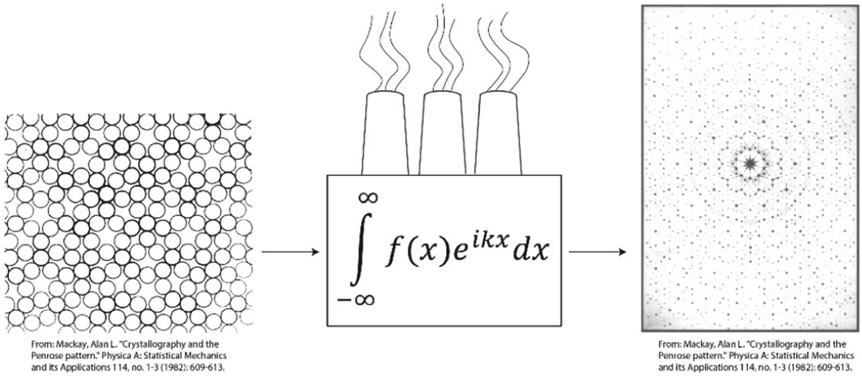

Fig. 2. Sphere packing of Penrose's Rhombus Tiling and subsequent diffraction pattern (Note: diffraction image was color inverted)

atmosphere. (Patel, 2017) A key challenge associated with the design of effective surface diffusers is making sure that sound energy is dispersed uniformly throughout the sound field while keeping the sound energy within the room and not absorbing it, thus maintaining an appropriate reverberation time. (Kuttruff, 2014) The diffusion of sound is enhanced when every position in the diffuse sound field receives the same density of reflected energy. A common practice among diffuser designers is to repeat a limited number of basic shapes on a periodic basis. (Ermann, 2015) This results in repetitive energy loops that reduce the diffuser's ability to uniformly disperse sound. (Tyler, 2016).

2.4 Schroeder Diffusers

In the 1970s, Manfred Schroeder (1975), utilized number theory to determine the optimal diffusion for a surface profile. A Schroeder diffuser consists of a series of wells of the same width and depth and can be arranged in a single or multi-dimensional configuration (see Fig. 3), and since their introduction, Schroeder's diffusers have been widely adopted in technical and architectural acoustics. (Cox and D'Antonio, 2003) The performance of Schroeder diffusers is limited by their ability to respond to the targeted sound wavelength, and depends on their surface design, arrangement, and depth profile. Aesthetically, Schroeder's irregular well arrangements can present a challenge for designers, and their protruding profiles make these products limiting for many wall applications. Contemporary architecture faces an important challenge in the absence of innovative acoustic designs that is complementary to established workflows. (Peter and Nguyen, 2021) A number of Architects are researching and developing new strategies overcome or complement the shortcomings of Schroeder diffusers through broadband absorption using Helmholtz resonators and Acoustic Metamaterials. (Setaki et al., 2016; Kladeftira et al., 2019; Nguyen et al., 2022; Cop, Nguyen, and Peters, 2023).

2.5 The Wieringa Roof

In 1981, the mathematician N. G. de Bruijn wrote a seminal paper on the algebraic theory and construction of Penrose in which he showed that they can be obtained from a periodic

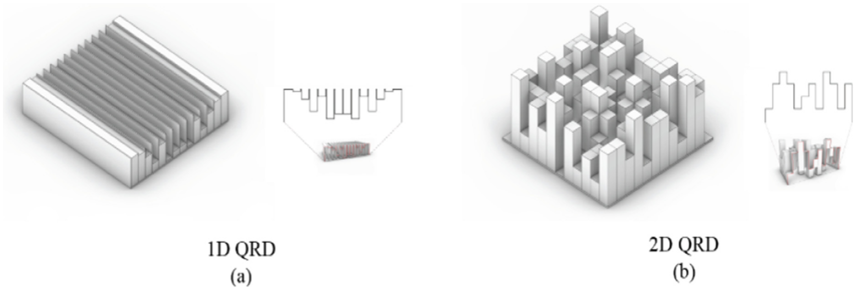

Fig. 3. Two types of Schroeder Quadratic Residue Diffusers (QRD)

5D space. (De Bruijn, 1981) Additionally, an observation made by his colleague R. M. A. Wieringa states that Penrose's Rhombi tiling can also be viewed as an orthogonal projection of a 3D surface containing a single rhombus (Fig. 4). De Bruijn named this surface a "Wieringa Roof" (WR) as he thought it could be used in architectural settings, particularly ceilings. (De Bruijn, 1981) Using an indexing system based around his construction of Penrose's P3 tiling, de Bruijn details a method to align and orient the rhombi of this 3D surface. This paper uses a similar method of indexing to orient and align the rhombi that are raised from 2D to 3D, as described by mathematician, and historian, Marjorie Senechal (1996). It is important to note that a WR is not a 3D tessellation of space, but the lifting of a planar tiling into a third dimension. The research described in this paper develops a parametric model for the WR that results in a generalization of the morphology of the surface. Due to this generalization, we designate any surface that can be orthographically projected from space to a plane to form Penrose's Rhombus tiling to be henceforth referred to as a Wieringa Surface (WS).

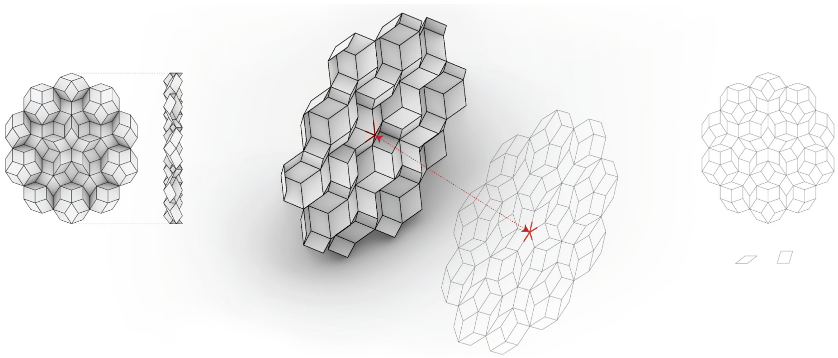

Fig. 4. Wieringa Roof morphology and projection to Penrose's P3 Tiling

2.6 Wieringa Surface for Architectural Acoustics

The application of aperiodicity into architectural acoustics is a promising endeavor, as aperiodic formations appear to have an advantage over periodic structures. (Karaiskou et al., 2020) Using a single asymmetric base shape, optimized for acoustical performance, D'Antonio and Cox (2017) have demonstrated curved diffusers that minimize the effects of periodicity. They reported that it is possible to increase the repeat distance, and hence improve performance, all the while creating a novel aesthetic for architects. In the design of Rafael Moneo's L'Auditori Barcelona, the Fibonacci sequence, an instance of 1D aperiodicity, was implemented to provide diffraction that progresses from low to high frequencies. The implementation of this sequence as done to avoid echoes and coloration. (Arau-Puchades, 2012) In 2018, Lee et al. verified that Penrose-tiling-type diffusers have only a very small dependence of incidence angle on scattering coefficients, which suggests that while there is promise for 2D aperiodic scattering surfaces, 3D aperiodic surfaces should be investigated. Rima Ajlouni (2018) has proposed that "quasi-periodic geometry" could be used for designing better surface diffusers as this means periodicity is removed from all directions and a limited number of manufactured shapes are used. An inadequacy of Rima's work is the lack of any acknowledgement or explanation for foundational governing mathematics that yields aperiodic forms and approaches the topic through deconstructing existing patterns found in historical documents.

Mackay's recognition that a clear diffraction pattern can result from the Fourier Transform of a sphere packing of the vertices of Penrose's Rhombus tiling, despite its lack of translational symmetry is a mathematical indication that there is an underlying order to its aperiodicity. The fact that aperiodic tessellations can be formulated from higher dimensional periodic spaces demonstrates that disorder and randomness are not the governing principles of aperiodic patterns. As mathematical models can explain this order (Fig. 5), predictions can be made on the implementation of aperiodicity into acoustics. We hypothesize that Wieringa Surfaces can provide a non-repeating scattering surface, perhaps comparable to the celebrated Schroeder QRDs, on the basis that its morphology lacks periodicity but is not random, a clear diffraction pattern results from similar geometries, its aperiodicity can be mathematically modelled from higher dimensional periodic spaces, and that its aesthetics are desirable for architectural implementation.

$$\sqrt{\frac{2}{5}}\begin{bmatrix} 0 & \sin(\frac{2\pi}{5}) & \sin(\frac{4\pi}{5}) & -\sin(\frac{4\pi}{5}) & -\sin(\frac{2\pi}{5}) \\ 1 & \cos(\frac{2\pi}{5}) & \cos(\frac{4\pi}{5}) & \cos(\frac{4\pi}{5}) & \cos(\frac{2\pi}{5}) \\ 1 & \cos(\frac{4\pi}{5}) & \cos(\frac{2\pi}{5}) & \cos(\frac{2\pi}{5}) & \cos(\frac{4\pi}{5}) \\ 0 & \sin(\frac{4\pi}{5}) & -\sin(\frac{2\pi}{5}) & \sin(\frac{2\pi}{5}) & -\sin(\frac{4\pi}{5}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

Fig. 5. The rotational matrix used in the construction of Wieringa Surfaces

3 Methods

In view of the complexity of the geometry, it was decided that the best approach would be to create a parametric model for quick surface iterations while using digital simulation to study the acoustic performance of the structure. A total of twelve case iterations, based off six parameters (Fig. 6), were used as the basis for testing and fabricating Wieringa surfaces. The parameters were: lattice size, α (alpha), β (beta), γ (gamma), δ (delta), and index order. The selected iteration for fabrication had a lattice size of 2, α = -0.25, β = -0.25, γ = -0.25, δ = 0.5, and had an indexing order of 1,2,3,4. For fabrication, the objective was to preserve aesthetic values while indicating a significant departure from the canonical Wieringa surface, as well as displaying all possible variations of rhombi orientations. The fabrication and testing required the model to be scaled to obtain a desired height, width, and radius that could perform acoustically. The fabricated model spans $20.76'' \times 20.73'' \times 4.45''$, the deepest well depth being $4''$. It was determined that computer numerically controlled (CNC) milling would provide the best surface fidelity since it contains a minimum amount of surface difference from the 3D model.

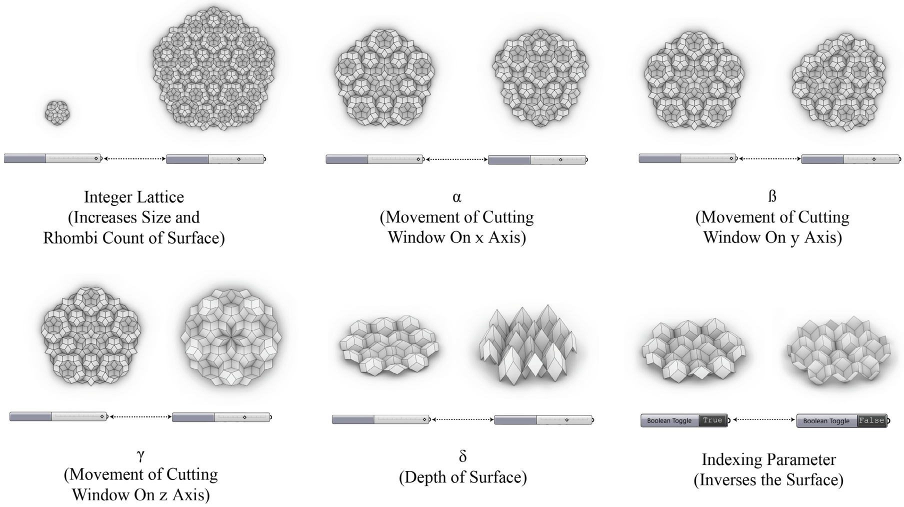

Fig. 6. Six parameters of the Wieringa Surface

3.1 Parametric Model

The parametric model developed in this research paper is based on the Cut and Project method. (Whittaker and Whittaker, 1988; Levine and Steinhardt, 1986; Socolar and Steinhardt, 1986) This method is well-developed in mathematics and essentially involves capturing a slice of a higher dimensional periodic space through a constructed cutting window and projecting the slice into a lower dimensional space, resulting in an aperiodic tessellation. The higher dimensional space used in this parametric model was a

5D periodic integer lattice, while the cutting window was a rhombic triacontahedron. Throughout extensive background research no evidence of a publicly available parametric CAD script was found. As of October 14, 2022, a public Wolfram Demonstration Project by Adam P. Goucher is available which can create a single canonical Wieringa surface arrangement with 5 degrees of sizing, however the resulting surface is small and cannot be converted to any CAD format. Likewise, there are several parametric tiling programs that allow for a P3 Penrose tiling to be constructed, however these programs are primarily for visual presentations and are not readily available to be placed within a CAD software. There exists a grasshopper plugin named Parakeet (Mottaghi and Khalil-Beigi, 2022) which can produce a P3 Penrose tiling, however the plugin only offers a single patch of Penrose's Rhombus tiling and does not allow for exploration of other portions of the tiling. Additionally, the plugin has no way of indexing the vertices of the tiling in order to systematically lift and orient the vertices into a third dimension, allowing for the construction of a Wieringa Surface.

3.2 Simulation

The software package AFMG Reflex (AFMG, 2022, October 13) was used to test the acoustic scattering properties for several Wieringa surface case studies throughout this investigation (Fig. 7). AFMG Reflex was selected on the basis of its simulative accuracy and informative results as demonstrated by Shtrepi et al. (2020) and Ajlouni (2018). A rigid two-dimensional slice of a three-dimensional object is subjected to the Boundary Element Method (BEM) for the calculation of acoustic diffusion and scattering coefficients, and modelled in an environment outlined and stipulated by the ISO 17497–1 Standard. Using BEM, it is possible to model acoustic performance on the basis that the linear partial differential equations used within the successive process, mathematically describe the wave-like phenomenon. (Cox and D'Antonio, 2009) With a simple width and height value, geometry is created natively within the program. The Wieringa Surfaces were measured in accordance with the modelling capabilities of the program using slices taken from each iteration. In this experiment, a Wieringa surface was subjected to incoming sound at a 45-degree angle with respect to the surface. Although the AFMG Reflex software is limited to two dimensions, it provides a valuable understanding of how a particular surface scatters sound. (Wolfgang and Feistel 2011) We recognize that the AFMG Reflex software is limited to two dimensions and therefore has limitations on how its results can be translated to three-dimensional scattering performance; however, our simulation results do indicate that the Wieringa surface may produce sound scattering and that this performance might be controlled through tuning its geometric properties.

3.3 Digital Fabrication

CNC milling was used as it was the most sensible way of creating a mold that could then be cast using a concrete-like material "Rockite." The material Rockite produces a smooth finish that will reflect most sound waves unlike wood or plastics and isolates the scattering performance away from sound absorptive properties. In the selected iteration, a block with dimensions of $22.5'' \times 22.5'' \times 5''$ represented the material stock to be

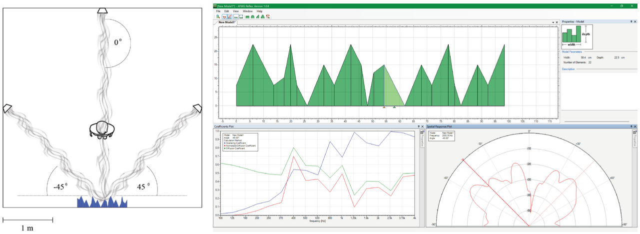

Fig. 7. AFMG Reflex simulation setup and result output

milled. A negative mold was produced into which the Rockite surface would be cast. The plugin RhinoCAM v. 2020 was used to simulate and aid the milling process (Fig. 8a). It was determined that white Styrofoam would be the most economical, convenient, and easy to dispose of material for the mold. Taking advantage of the foam's lighter weight, it was taped to the CNC bed for stability and tests were performed on feeds and speeds. The first pass of the CNC mill was a horizontal roughing at 14000 RPM with a 0.5″ square carbide end mill in the clockwise direction and was controlled at a step over rate of 90%. Feed rates varied between 350–500 in/min. A second pass was then performed to clear the flat tops with the same speed and feed as the horizontal roughing. A 0.5″ ball carbide end mill was then used for parallel finishing at 14000 RPM. A speed of 300 in/min and a step-over control of 5% were used to achieve flat faces for the rhombi with minimal striations (Fig. 8b). For each block of Styrofoam, the total milling time resulted in approximately 4 h due to the low step-over rate in the parallel finishing, however this reduced post-process sanding (Fig. 8c).

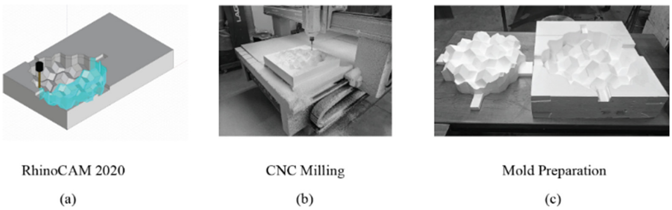

Fig. 8. Milling and processing of Styrofoam molds

4 Results

4.1 Acoustic

In total, twelve iterations of the Wieringa surface were developed using our parametric model. A 2D slice of the 3D geometry was taken at the centroid of each surface and modeled in AFMG Reflex to evaluate scattering properties. In the absence of a change in any other factors, an increase in the delta parameter (Fig. 6) improved acoustic performance for both normalized diffusion coefficients and scattering coefficients (Fig. 9a). This is expected as the increase in delta parameter increases the peaks and valleys of the Wieringa surface and thereby allowing for deeper and larger acoustic well depths to occur. This pattern is best observed when the simulated sound source was placed at 45 degrees to the normal. When alpha and beta parameters (Fig. 6) were altered while other variables are left constant there is an observable pattern in the graphs (Fig. 9b). Overall deviation between iterations is insignificant, but local variability between iterations exists.

Consequently, because orientation is not a factor in determining the acoustic performance of a WS, less planning would be devoted to the fabrication of these surfaces since both a positive and negative cast will produce similar acoustical results. This is a great advantage for mass production, as the cost of fabrication is normally proportional to the complexity of the surface's profile. Considering several iterations of the WS' and comparing them to the 1D and 2D Schroeder QRDs, it is evident that the WS' are significantly less effective at lower frequencies (Fig. 9c). For the normalized diffusion coefficient, both 1D and 2D QRDs outperform the WS' for $x < 400$ Hz, while for the scattering coefficient, QRDs outperform the Wieringa surfaces as much as $x < 500$ Hz. In spite of this, the WS' are comparable to calculable periodic surfaces at $x > 500$ Hz (See Fig. 9c). This would suggest that WS are not sufficient for acoustic scattering at frequencies $x < 500$ Hz, however at $x > 500$ Hz are comparable (and at some frequencies superior to) both forms of QRDs. As previously written, these simulations were carried out using two dimensional BEM techniques and therefore the results cannot be directly translated to the actual three-dimensional geometry of the Wieringa surface. However, we speculate that these results indicate that there might also be similar acoustic properties when these surfaces are simulated using three-dimensional software.

4.2 Fabrication Process

The material Rockite was selected for to its suitable properties to be casted and cured as a solid mass that can absorb negligible acoustic energy. However, working with concrete requires the understanding of physical limitations that must be overcome if there is to be functionality within the fabricated model. The first model was a complete negative out of a stock material, meaning that all the material removed from the stock material was to be replaced by Rockite as a positive molded cast (Fig. 10). In this first experiment, the milled white Styrofoam came out with crisp edges and flat rhombi due to the 5% step-over of the drill bit in the parallel finishing. This was unexpected as it was assumed that white foam would not have the rigidity to create a clean cast due to the forces exerted on it by the CNC mill. White Styrofoam produced a defined, rigid, and affordable mold was produced. A coating of Vaseline was applied accordingly as a releasing agent. The finishes on the model were smooth, the lines between the rhombi were crisp, there was no observable cracking, and the integrity of the model was strong. The only setback of the model was its extreme weight. The mold itself weighed 15.9 kg. Because our purposes of the Wieringa surface was acoustics, we knew that a solid mold cast of Rockite was not the final iteration in the fabrication process. With the insights that we had gained with the first cast it was decided that a compression mold would be the next experiment. A negative mold would be milled out and then a positive of that negative would also be milled, however the positive would be offset by $0.5''$. It was hypothesized that this would give a sufficient the wall thickness to uphold the complex surface but also be resilient to

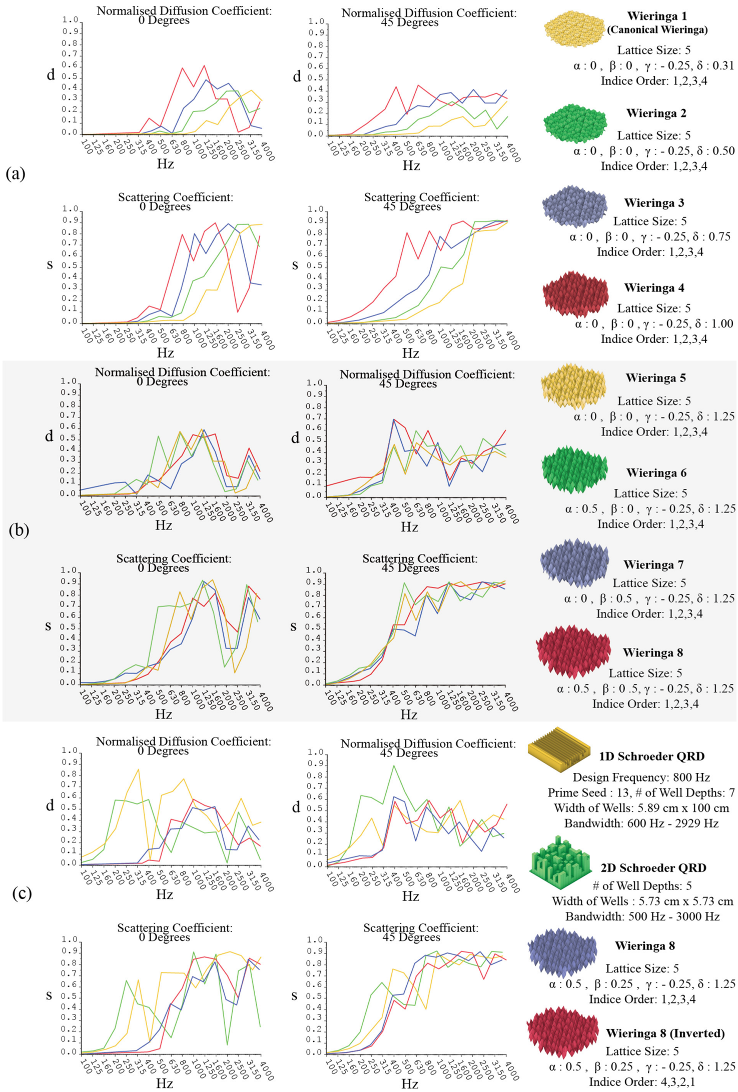

Fig. 9. Acoustic Results

handling, mounting, and small tensile forces placed upon on it. Guides were etched into both the positive and the negative molds in an effort to prevent uneven accumulation of material. While the Rockite mixture, Vaseline application, temperature, and curing time were all equal to the first cast, the second cast was much more difficult to release. Upon release there was noticeable air bubble pockets that had formed at the valleys of the surface which created unwanted aesthetics, flaking, and cracking. It was hypothesized that there was not enough release area within the initial compression of the mold to allow air to escape. Additionally, it was noted that agitating the bass of the mold would be an added part to the fabrication process. Despite these drawbacks, the mold itself was significantly lighter then the first mold, all the while keeping its strength integrity. This allowed us to produce a third mold which we kept the idea of a compression mold, while addressing issues of air bubbles and the release of those bubbles. Fortunately, with the added agitation to the mixture placed in the mold, a redesigning of releasing areas, and a well-developed casting routine, our third iteration was a success. Air bubbles had not created the unwanted aesthetics, flaking, and cracking. Additionally, both the structural integrity and the molds fidelity were retained, thereby establishing a clear lineage of experimentation that ultimately led to a successful prototype (Fig. 10).

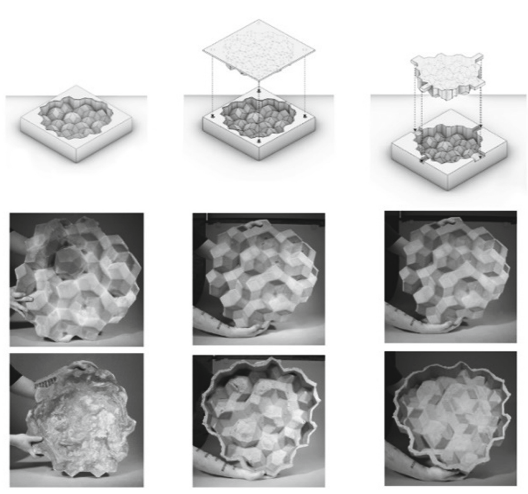

Fig. 10. Diagram Explaining Molding Process Discussion

5 Discussion

The formation of a parametric model of the Wieringa surface is an exciting step in design and is a reaffirmation of the intimate relationship that mathematics has with architecture. The age of computation, along with the democratization of mathematical knowledge and tools can now allow us to probe into spaces once deemed unobservable. There are many opportunities for designers to investigate properties and aspects of 5D space with the Wieringa Surfaces and based on the acoustic findings, as well as a freely distributed grasshopper script, we conjecture that they will be beneficial to structures, aesthetics, and acoustics as de Bruijn had once hoped. Beyond the mathematical contributions, we have investigated the acoustic properties of these surfaces through simulation. Due to the scope of this experiment, initial acoustic analysis was limited to 2D analysis and provided ideas as to what can be achieved with the now parameterized Wieringa Surface. While there is confidence in the initial results through AFMG Reflex, it is noted that the

Wieringa Surface is a complicated 3D surface topology. Further research implementing physical testing will need to be conducted to verify results obtained. Additionally, assessments of sound quality within an architectural setting will be required to fully test the postulated acoustic application. While a concrete molding process was investigated in this paper it is not limited to this fabrication option for architectural acoustics. Although not implemented in the fabrication process of this paper, the use of modularity and aggregate assembly has been built into the code for future experimentation and fabrication. The use of Styrofoam as a material is destructive, alternatively, a nondestructive option for molding would involve silicone casting and release. This would be beneficial in a fabrication setting that calls for precise repeatability and mass production. It is important to note that while Wieringa surfaces have an uncountably infinite number of morphologies, there are other aperiodic geometries that extend beyond the plane that can also be explored and exploited (Fig. 11).

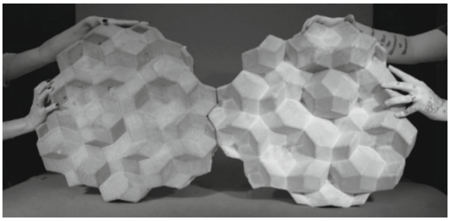

Fig. 11. Casted Tile B and A interlocking, respectively

6 Conclusion

The formation of a parametric model of the Wieringa Surface is an exciting step in design and provides opportunities for the exploration of new architectural knowledge. We have discussed the mathematical background to provide other architectural designers with the requisite information to holistically explore these amazing surfaces as well as provide a rational foundation for the hypothesis that such surfaces have useful scattering and acoustic properties. The acoustic simulations conducted revealed that Wieringa Surfaces have the potential for tunable acoustic performance when properly optimized. Our Fabrication experiments were performed to demonstrate avenues of physical construction and knowledge gained from these experiments have been outlined for future refinement. Wieringa Surfaces are a testament to the intimate relationship that mathematics has with architecture, acoustics, and design, and we conjecture that these surfaces will be beneficial to structures, aesthetics, and acoustics as de Bruijn had once hoped.

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